



Sem 1 Examination 2010

Question/answer booklet

MATHEMATICS: SPECIALIST 3CDMAS

Section Two (calculator-assumed)

Name: _____

Teacher: _____

Time allowed for this section

Section Two

Reading time before commencing work: 10 minutes

Working time for paper: 100 minutes

Material required/recommended for this paper

To be provided by the supervisor

Question/answer booklet for Section Two and a formula sheet (from Curriculum Council) which was provided with Section One.

To be provided by the candidate

Section Two:

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, and up to three calculators, (CAS, graphic or scientific), which satisfy the conditions set by the Curriculum Council for this course. Up to two (2) A4 pages of notes (handwritten, photocopied or typed on both sides) that may be either personally or commercially produced. *It must not be folded, have anything stuck to it or have correction fluid/tape on it.*

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|--|-------------------------------|------------------------------------|----------------------------------|-----------------|
| Section One: Calculator-free | 6 | 6 | 50 | 39 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 79 |
| | | | | 118 |

Instructions to candidates

- The rules for the conduct of Western Australian examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.
- Answer the questions according to the following instructions.

Section Two: Write answers in this Question/Answer Booklet. **All** questions should be answered.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil** except in diagrams.

- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

MARK ALLOCATION AND RECORDS:

| Section | Question | Marks | Awarded |
|---------|------------|-----------|---------|
| TWO | 7 | 4 | |
| | 8 | 4 | |
| | 9 | 5 | |
| | 10 | 5 | |
| | 11 | 5 | |
| | 12 | 5 | |
| | 13 | 7 | |
| | 14 | 10 | |
| | 15 | 10 | |
| | 16 | 11 | |
| | 17 | 13 | |
| | | Penalties | - 1/2/3 |
| | TWO | 79 | |

| | |
|-----------|--|
| Penalties | |
|-----------|--|

| | |
|---------------|--|
| Rounding (-1) | |
|---------------|--|

| | |
|------------|--|
| Units (-1) | |
|------------|--|

| | |
|---------------|--|
| Notation (-1) | |
|---------------|--|

Section Two: Calculator – 79 marks

This section has **eleven (11)** questions. Attempt **all** questions.

Working time: **100 minutes**

7. [4 marks]

If $z = \text{cis } \theta$, show that $\frac{z - z^{-1}}{i(z + z^{-1})} = \tan \theta$.

8. [4 marks]

Consider that the complex number z is a function of time t

i.e. $z = x(t) + iy(t)$. Prove that $\frac{d}{dt} (\text{Re } z) = \text{Re} \left(\frac{dz}{dt} \right)$.

9. [5 marks]

The locus of points z in the complex plane is determined by the constraint $z\bar{z} + z + \bar{z} = 24$. Demonstrate how to find the nature of this locus.

10. [3, 2 marks]

If $z = 4 \operatorname{cis} \theta$ and $z = A \operatorname{cis} \left(\frac{2\pi}{3} \right)$ are two of the four, fourth roots of z , find:

(a) exactly, all values of A and θ , where $-\pi \leq \theta \leq \pi$.

(b) z in exact polar form

11. [3, 2 marks]

Separately sketch the following in the Argand plane.

(a) $|1 - z| < |z + i - 2|$

(b) $\operatorname{Arg}(z + 1) \geq \frac{\pi}{4}$

14. [3, 1, 3, 3 marks]

(a) Show that $(z - 1 - i)$ is a factor of $z^3 - 4z^2 + 6z - 4$.

(b) Write another complex factor of the cubic expression.

(c) From parts a) and b) find a quadratic factor of the cubic expression.

(d) Show how to find the last factor of the cubic expression without the use of your calculator.

15. [2, 3, 2, 1, 2 marks]

If $\mathbf{u} = \langle -2, 3, 1 \rangle$ and $\mathbf{v} = \langle 3, 1, -5 \rangle$ find:

(a) $4\mathbf{u} - 3\mathbf{v}$

(b) the size of the angle between \mathbf{u} and \mathbf{v}

(c) $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$

(d) the acute angle between \mathbf{v} and the x axis

(e) the acute angle between \mathbf{u} and the $x - y$ plane.

17. [3,3,3,4 marks]

Solve each equation below for z without using your calculator.

(a) $z^4 + 6z^2 + 5 = 0$

(b) $(z + 5 + i)^2 = -9$

(c) $3z - 4\bar{z} = 5 + 14i$

(d) $z + \frac{8}{z} = 4$
